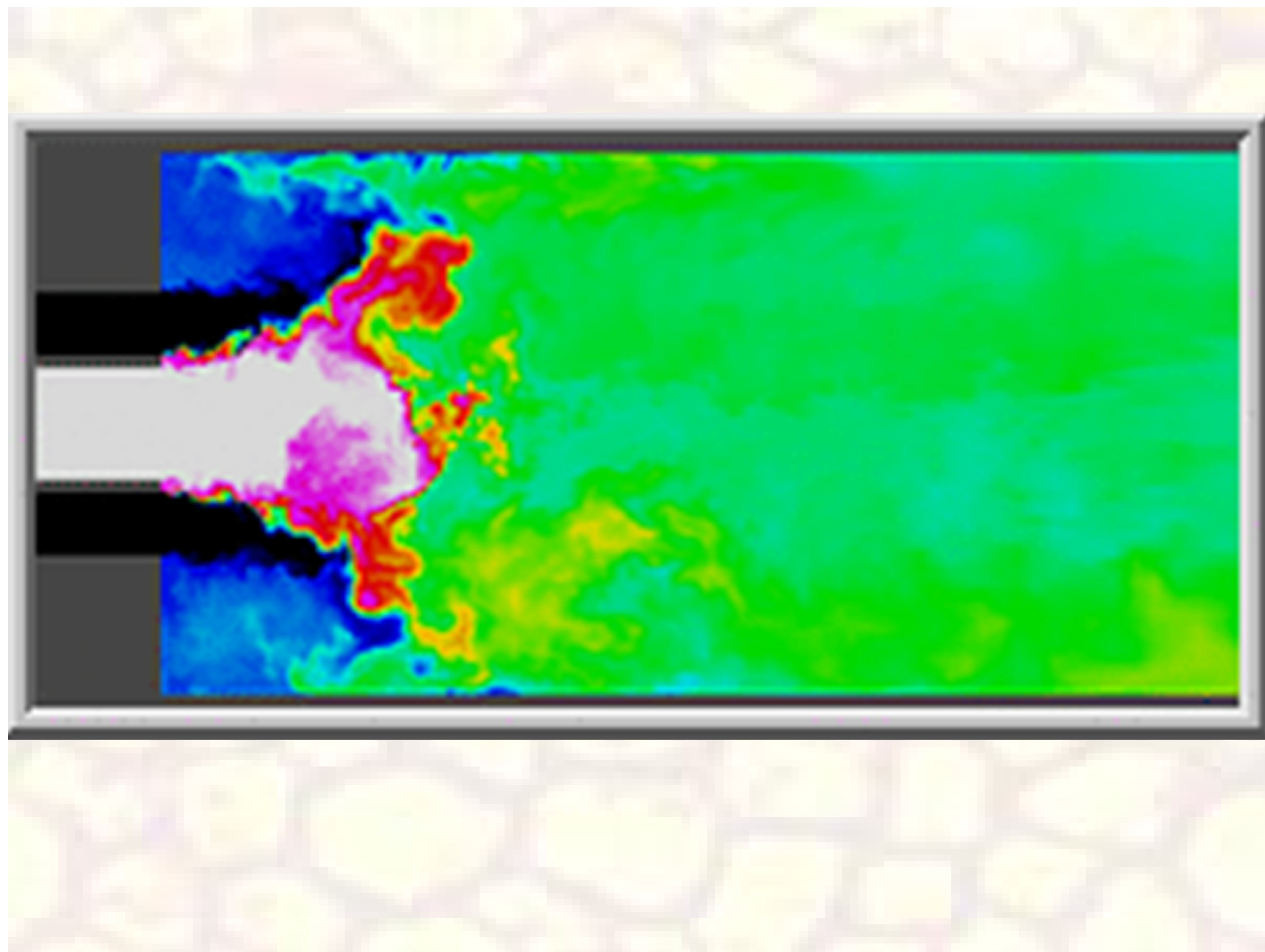


Fluid Properties

Viscosity of Fluid



2.1 Dynamic Viscosity

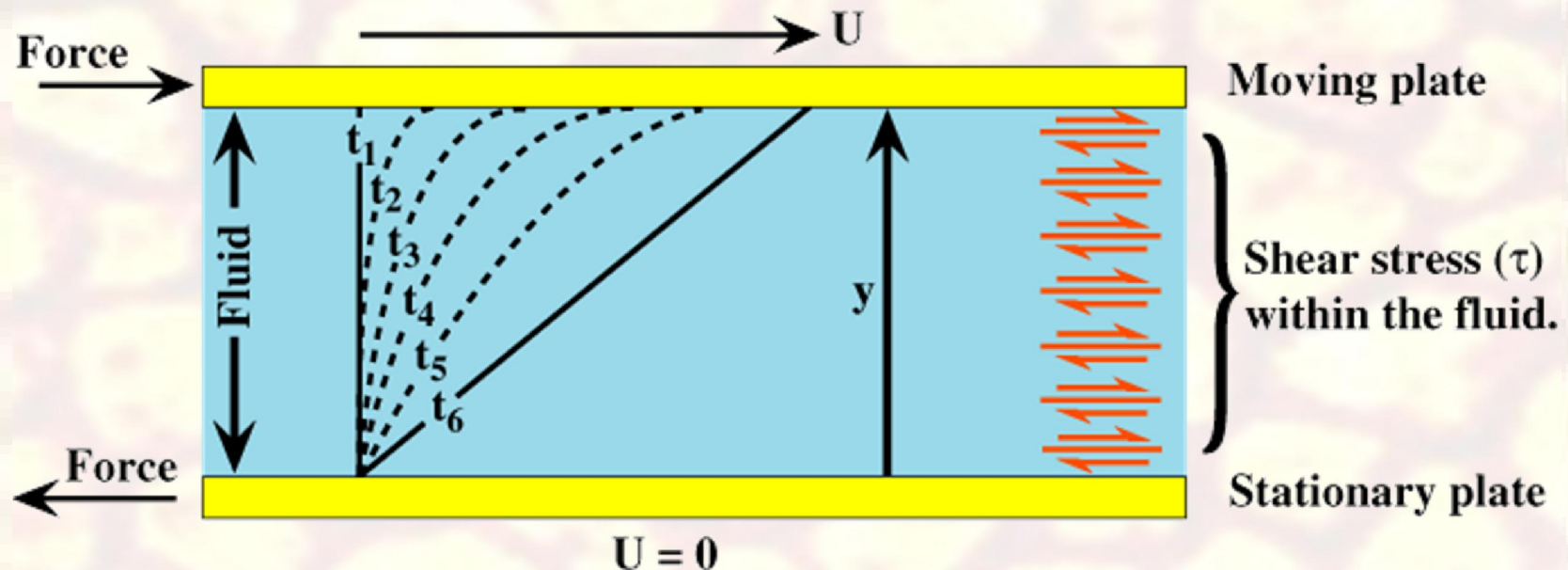
- As a fluid moves, a shear stress is developed in it, the magnitude of which depends on the viscosity of the fluid.
- *Shear stress*, denoted by the Greek letter (tau), τ , can be defined as the force required to slide one unit area layer of a substance over another.
- Thus, τ is a force divided by an area and can be measured in the units of N/m^2 (Pa) or lb/ft^2 .

Fluid flow between two parallel plates

The bottom plate is fixed and the top plate is accelerated by applying some force that acts from left to right.

The upper plate will be accelerated to some terminal velocity and the fluid between the plates will be set into motion.

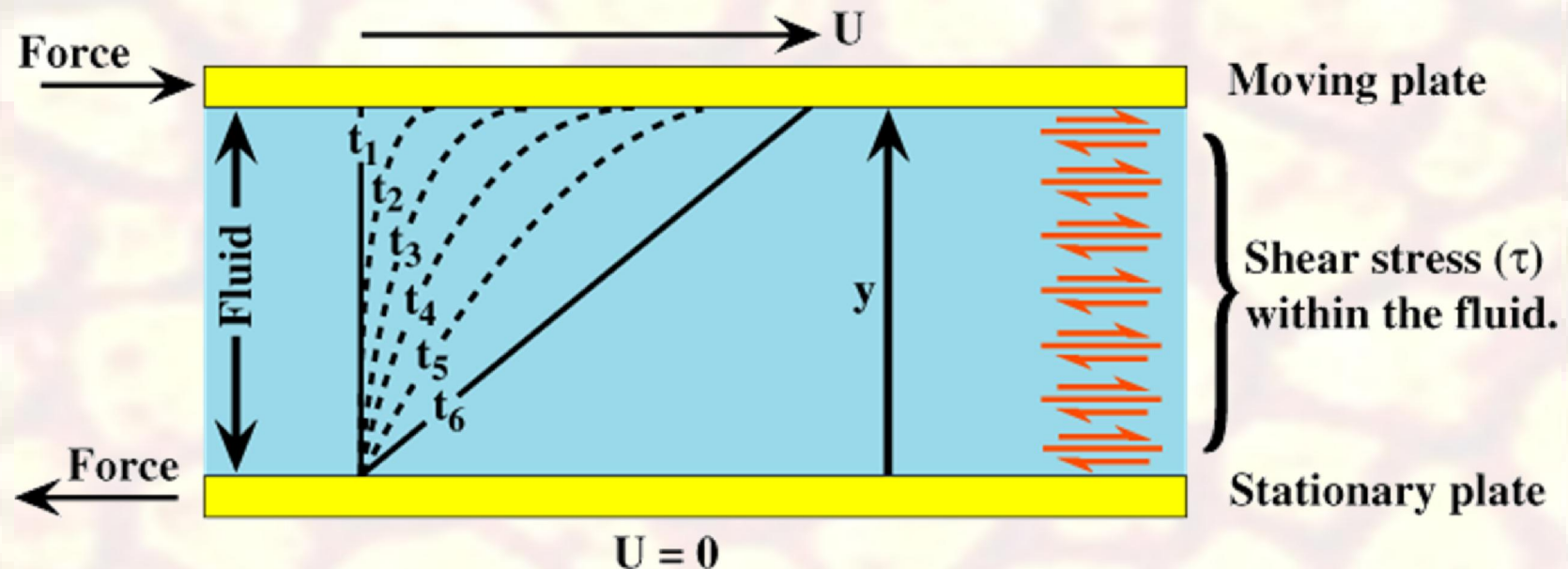
Terminal velocity is achieved when the applied force is balanced by a resisting force (shown as an equal but opposite force applied by the stationary bottom plate).



As the upper plate begins to accelerate the velocity of the fluid molecules in contact with the plate is equal to the velocity of the plate (a *no slip condition* exists between the plate and the fluid).

Fluid molecules in contact with those against the plate will be accelerated due to the viscous attraction between them... and so on through the column of fluid.

The viscosity of the fluid (μ , the attraction between fluid molecules) results in layers of fluid that are increasingly further from the moving plate being set into motion.

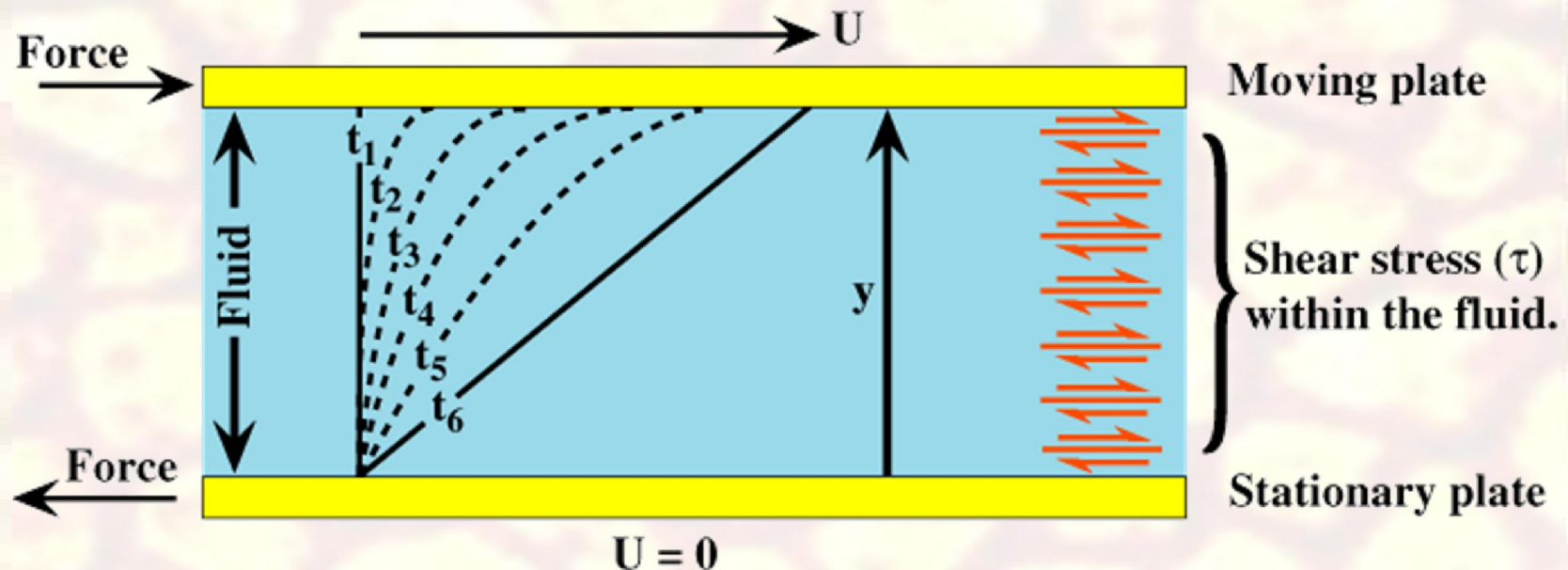


What is this resisting force? It is “fluid resistance” (μ) rather than a force applied by the lower plate (viscosity is often called “fluid friction”).

Note that as the velocity increases upwards through the column of fluid, there must be slippage across any plane that is parallel to the plates within the fluid.

At the same time there must be resistance to the slippage or the upper plate would accelerate infinitely.

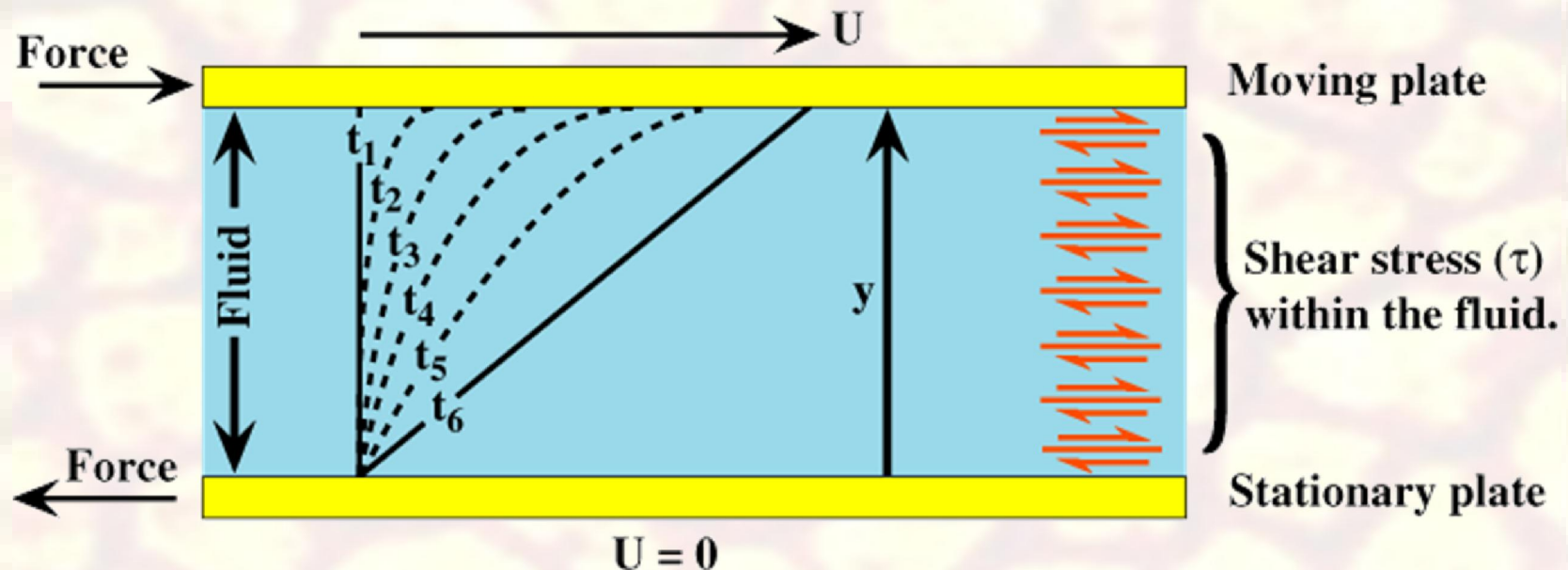
This same resistance results in the initial acceleration of every layer of fluid to its own terminal velocity (that decreases downwards).



Fluid viscosity is the cause of fluid resistance and the total viscous resistance through the column of fluid equals the applied force when the terminal velocity is achieved.

The viscous resistance results in the transfer of the force applied to the top plate through the column of fluid.

Within the the fluid this force is applied as a shear stress (τ , the lower case Greek letter tau; a force per unit area) across an infinite number of planes between fluid molecules from the top plate down to the bottom plate.

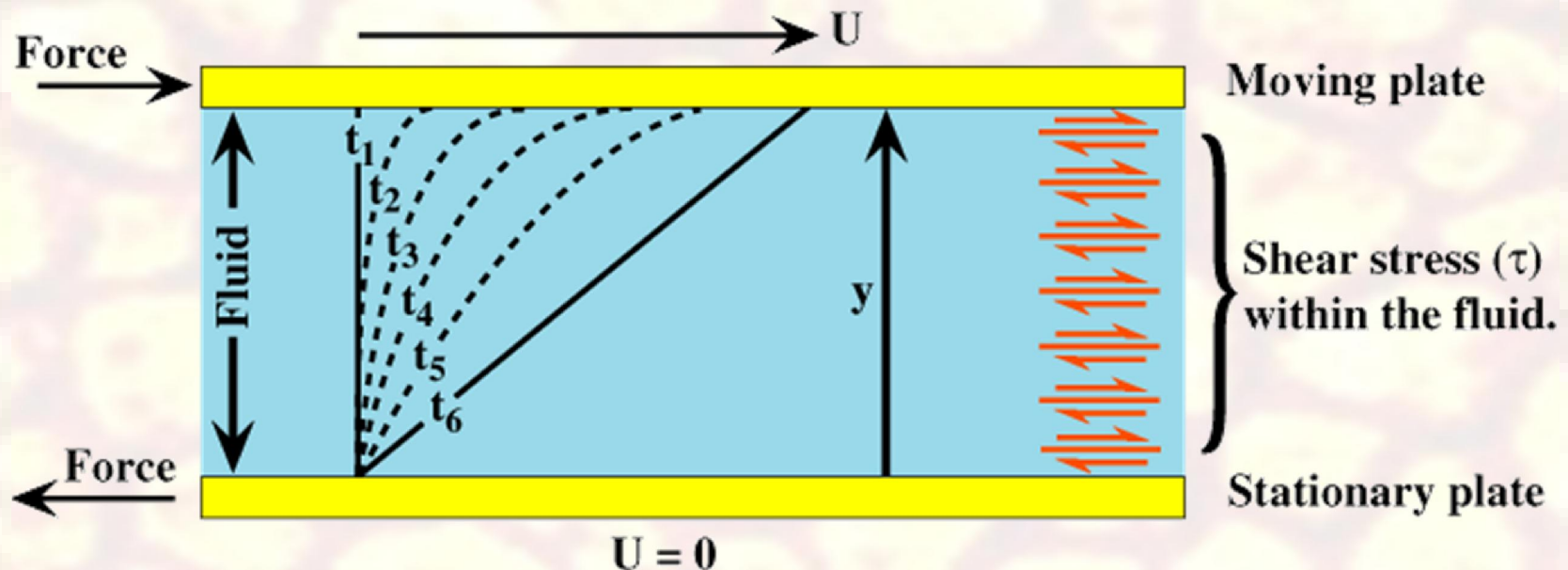


The shear stress transfers momentum (mass times velocity) through the fluid to maintain the linear velocity profile.

The magnitude of the shear stress is equal to the force that is applied to the top plate.

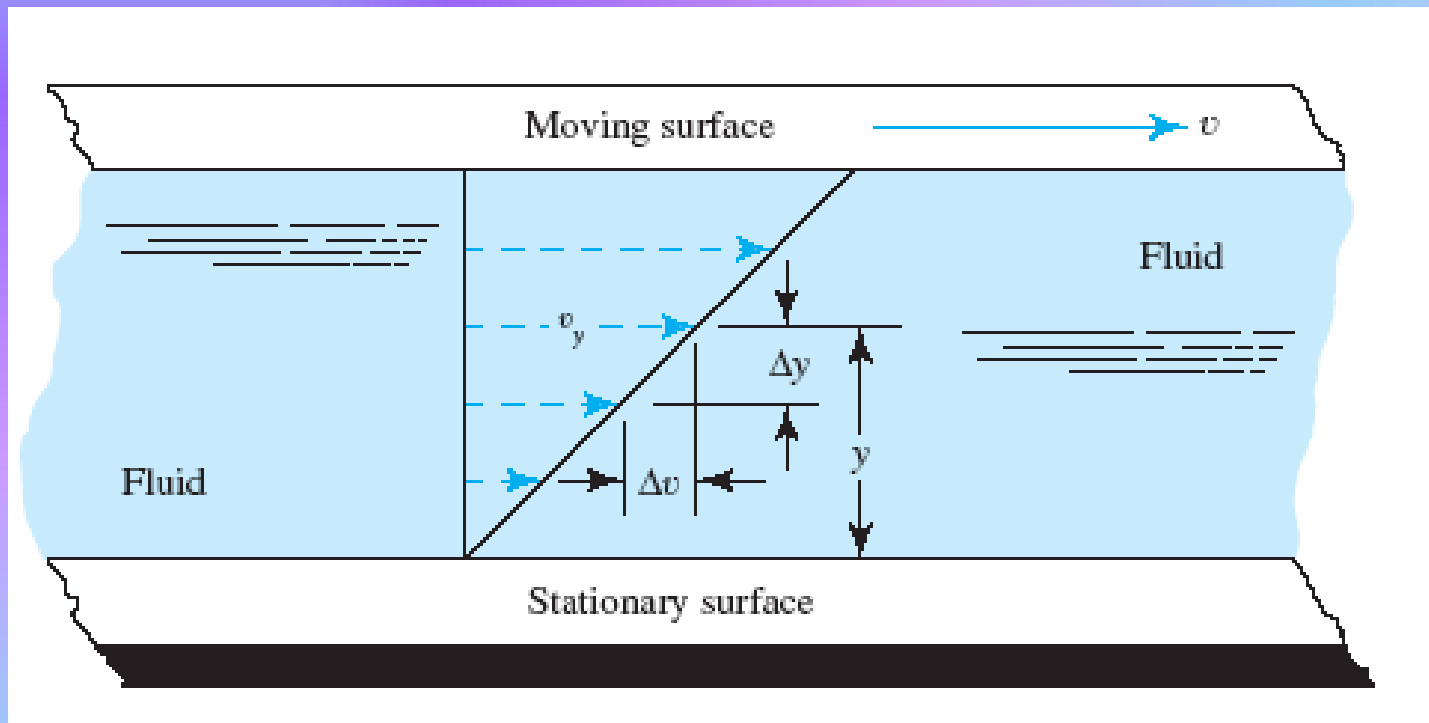
The relationship between the shear stress, the fluid viscosity and the velocity gradient is given by:

$$\tau = \mu \frac{du}{dy}$$



2.1 Dynamic Viscosity

- Fig 2.1 shows the velocity gradient in a moving fluid.



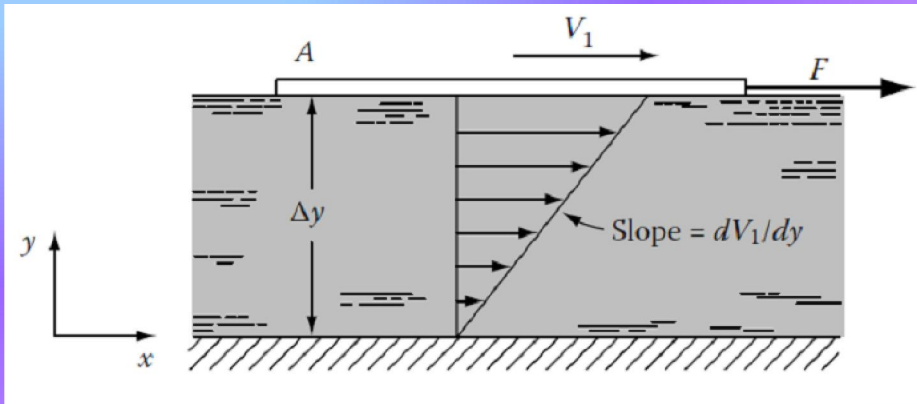
2.1 Dynamic Viscosity

- The fact that the shear stress in the fluid is directly proportional to the velocity gradient can be stated mathematically as

$$\tau = \mu(\Delta v / \Delta y) \quad (2-1)$$

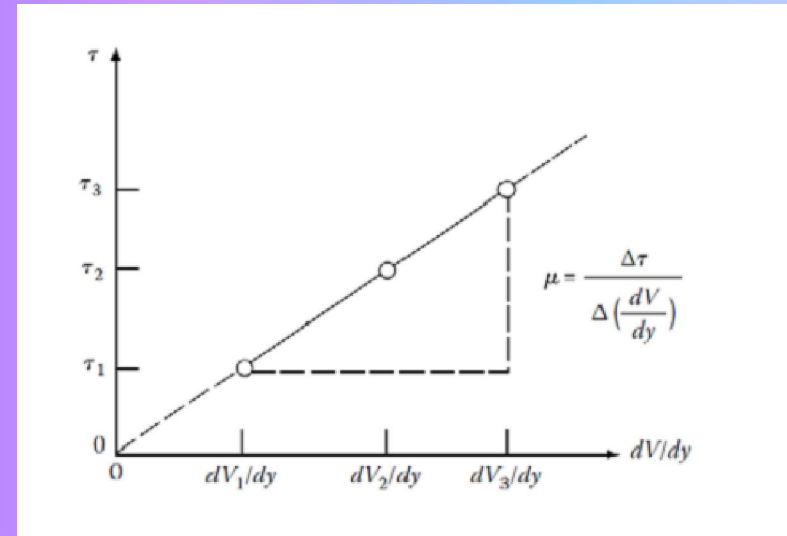
where the constant of proportionality μ (the Greek letter miu) is called the ***dynamic viscosity*** of the fluid. The term ***absolute viscosity*** is sometimes used.

2. Viscosity of Fluids



Shear stress applied to a fluid

If this experiment is repeated with F_2 as the force, a different slope or strain rate results: dV_2/dy . In general, to each applied force there corresponds only one shear stress and only one strain rate. If data from a series of these experiments were plotted as τ versus dV/dy



- A plot of τ versus dV/dy (a rheological diagram) for Newtonian fluids

In other words, viscosity indicates how a fluid will react (dV/dy) under the action of an external shear stress (τ).

2. Viscosity of Fluids

- Newton's law of viscosity states that the shear force to be applied for a deformation rate of (dV/dy) over an area A is given by,

$$F = \mu A (dV / dy)$$

or

$$F / A = \tau = \mu (dV / dy) = \mu (V / y)$$

$$\tau = \mu \frac{dV}{dy}$$

- F is the applied force in N, A is area in m^2 , dV/dy is the velocity gradient (or rate of deformation), $1/s$, perpendicular to flow direction, here assumed linear, and μ is the proportionality constant defined as the dynamic or absolute viscosity of the fluid.

2.1.1 Unit of Dynamic Viscosity

- The definition of dynamic viscosity can be derived from Eq. (2–1) by solving for μ :

$$\mu = \frac{\tau}{\Delta v / \Delta y} = \tau \left(\frac{\Delta y}{\Delta v} \right) \quad (2-2)$$

- The units for μ can be derived by substituting the SI units into Eq. (2–2) as follows:

$$\mu = \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}}{\text{m/s}} = \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

2.1.1 Unit of Dynamic Viscosity

- Because Pa is another name for N/m², we can also express μ as

$$\mu = \text{Pa}\cdot\text{s}$$

- Because 1N = 1 kg·m/s² can be expressed as

$$\mu = \text{N} \times \frac{\text{s}}{\text{m}^2} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2} \times \frac{\text{s}}{\text{m}^2} = \frac{\text{kg}}{\text{m}\cdot\text{s}}$$

- Thus N·m/s², Pa·s or kg/m·s may be used for μ in the SI system.

2.1.1 Unit of Dynamic Viscosity

- Table 2.1 shows the system conversion.

Unit System	Dynamic Viscosity Units
International System (SI)	$\text{N}\cdot\text{s}/\text{m}^2$, $\text{Pa}\cdot\text{s}$, or $\text{kg}/(\text{m}\cdot\text{s})$
U.S. Customary System	$\text{lb}\cdot\text{s}/\text{ft}^2$ or $\text{slug}/(\text{ft}\cdot\text{s})$
cgs system (obsolete)	$\text{poise} = \text{dyne}\cdot\text{s}/\text{cm}^2 = \text{g}/(\text{cm}\cdot\text{s}) = 0.1 \text{ Pa}\cdot\text{s}$ $\text{centipoise} = \text{poise}/100 = 0.001 \text{ Pa}\cdot\text{s} = 1.0 \text{ mPa}\cdot\text{s}$

2.2 Kinematic Viscosity

- The kinematic viscosity ν (the Greek letter nu) is defined as

$$\nu = \mu / \rho \quad (2-3)$$

- Because μ and ρ are both properties of the fluid, ν is also a property.
- We can derive the SI units for kinematic viscosity by substituting the previously developed units for μ and ρ :

$$\nu = \frac{\mu}{\rho} = \mu \left(\frac{1}{\rho} \right)$$

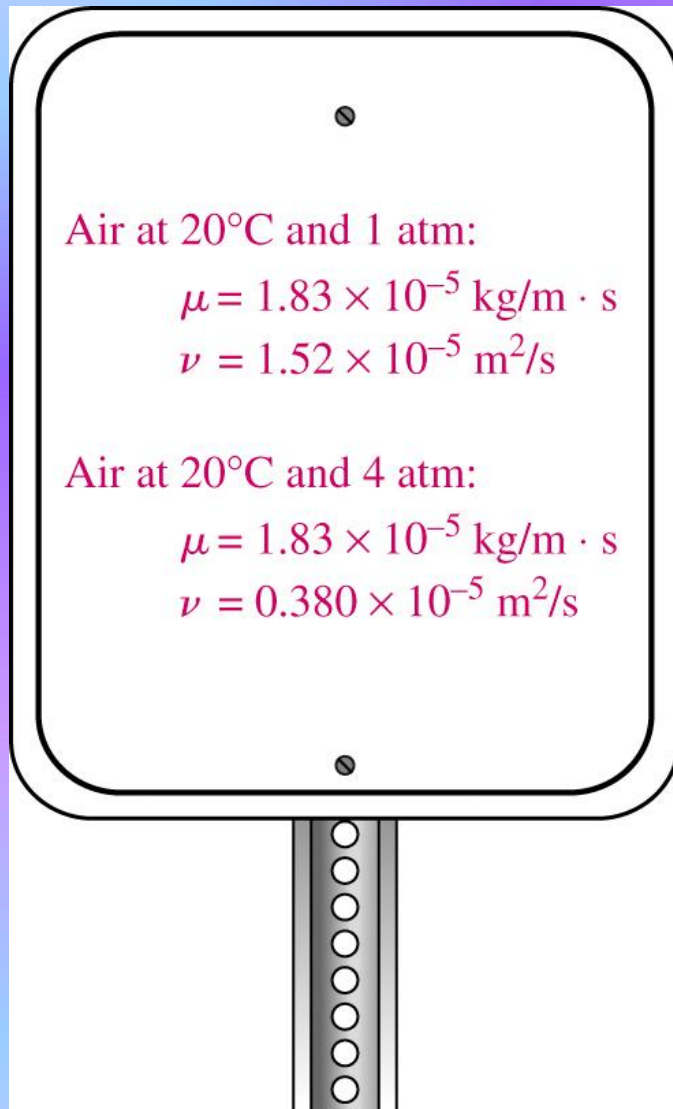
$$\nu = \frac{\text{kg}}{\text{m} \cdot \text{s}} \times \frac{\text{m}^3}{\text{kg}}$$

$$\nu = \text{m}^2 / \text{s}$$

2.2 Kinematic Viscosity

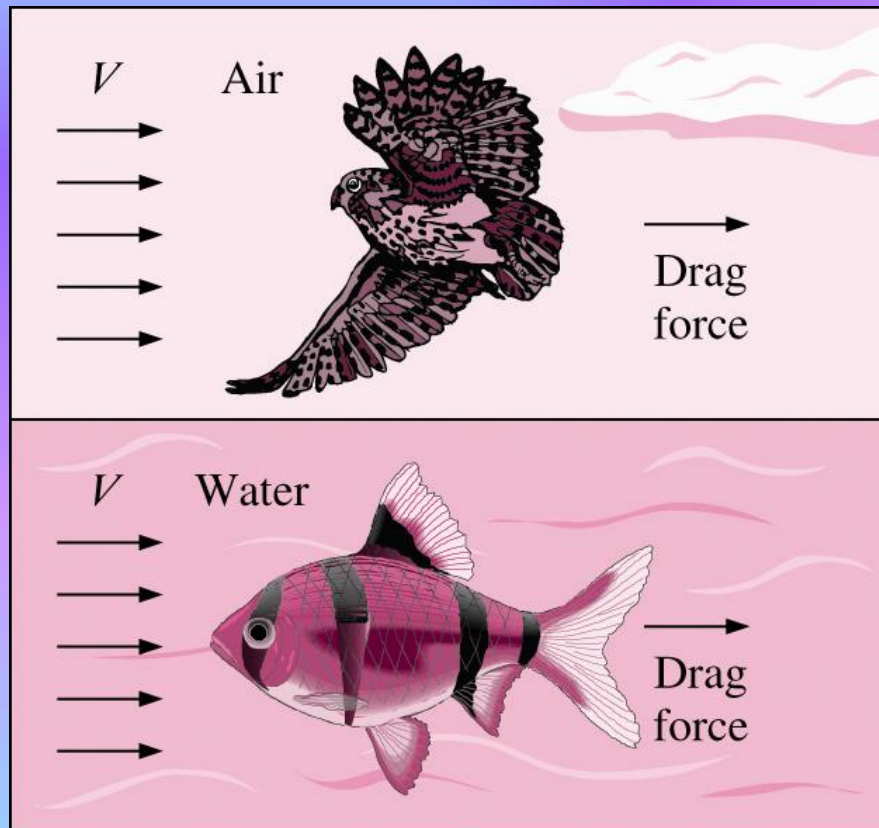
- Table 2.2 lists the kinematic viscosity units in the three most widely used systems.

Unit System	Kinematic Viscosity Units
International System (SI)	m^2/s
U.S. Customary System	ft^2/s
cgs system (obsolete)	$\text{stoke} = \text{cm}^2/\text{s} = 1 \times 10^{-4} \text{ m}^2/\text{s}$ $\text{centistoke} = \text{stoke}/100 = 1 \times 10^{-6} \text{ m}^2/\text{s} = 1 \text{ mm}^2/\text{s}$



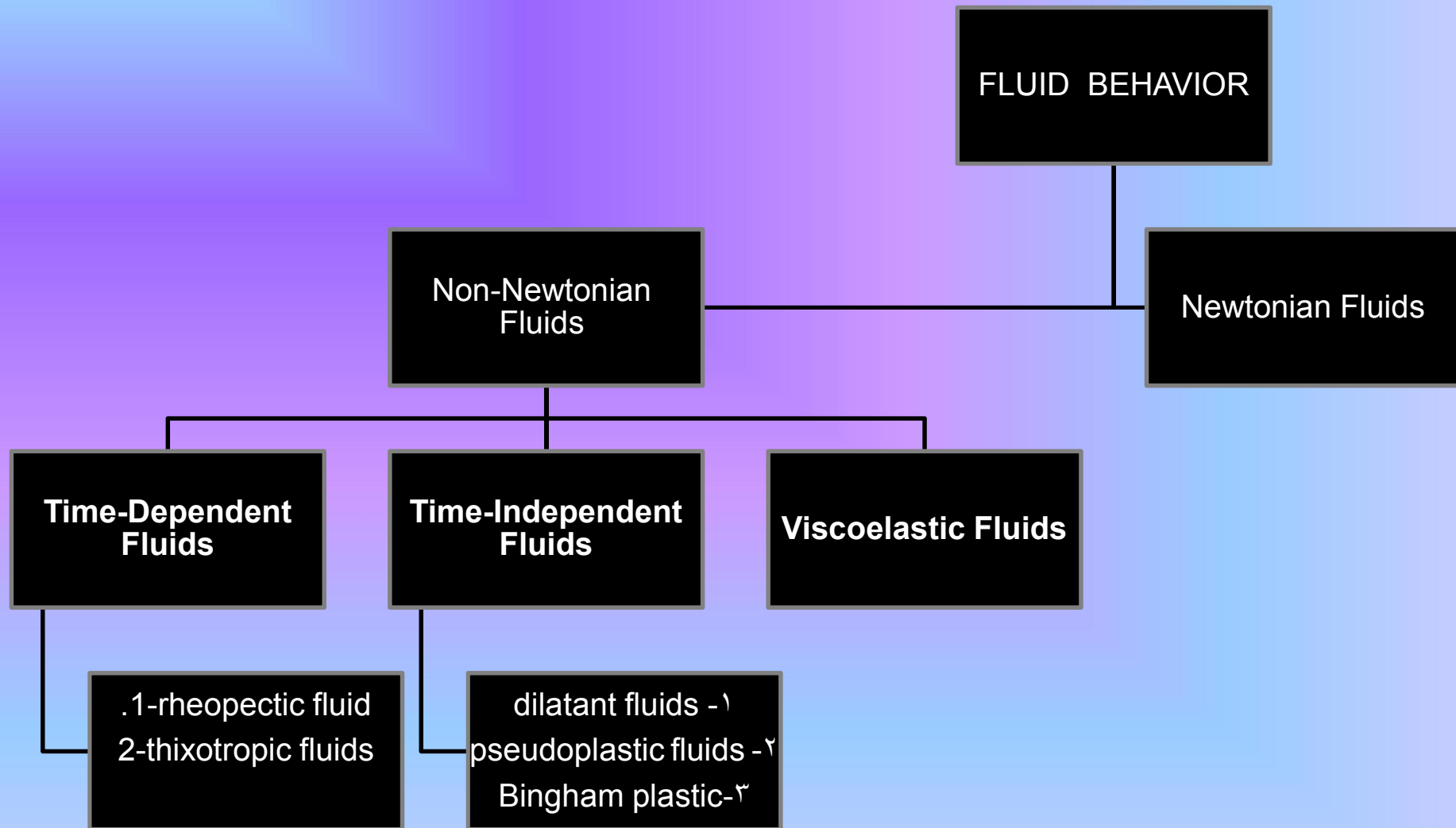
Dynamic viscosity, in general does NOT depends on pressure,

But Kinematic viscosity does.



- Viscosity is a property that represents the internal resistance of a fluid to motion.
- The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity.

2. Viscosity of Fluids

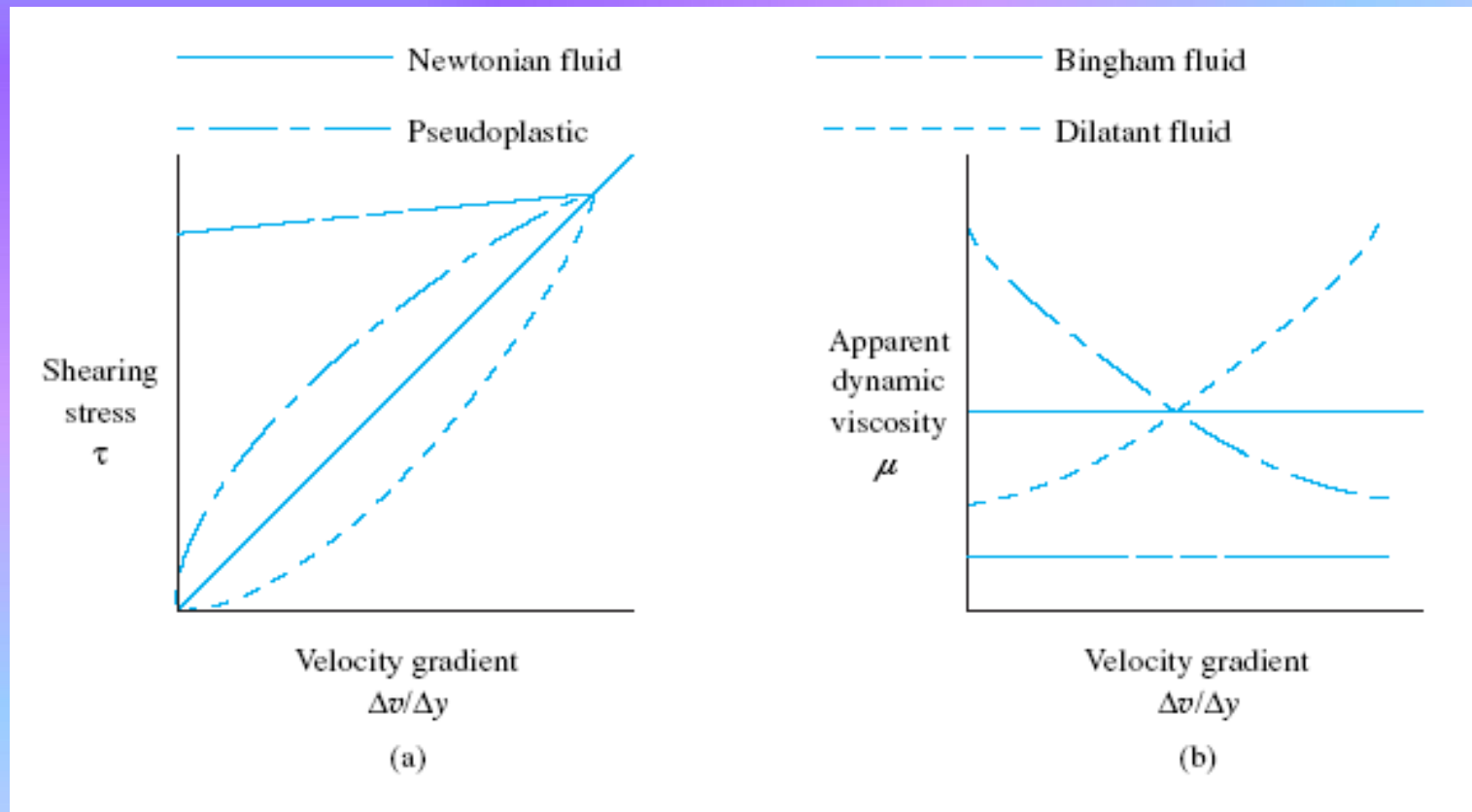


2.3 Newtonian Fluids and Non-Newtonian Fluids

- The study of the deformation and flow characteristics of substances is called *rheology*, which is the field from which we learn about the viscosity of fluids.
- One important distinction is between a *Newtonian fluid* and a *non-Newtonian fluid*.
- Any fluid that behaves in accordance with Eq. (2–1) is called a *Newtonian fluid*.
- Conversely, a fluid that does not behave in accordance with Eq. (2–1) is called a *non-Newtonian fluid*.

2.3 Newtonian Fluids and Non-Newtonian Fluids

- Fig 2.2 shows the Newtonian and non-Newtonian fluids.



2.3 Newtonian Fluids and Non-Newtonian Fluids

- Two major classifications of non-Newtonian fluids are *time-independent* and *time-dependent* fluids.
- As their name implies, time-independent fluids have a **viscosity at any given shear stress that does not vary with time.**
- The viscosity of time-dependent fluids, however, **changes with time.**

2. Viscosity of Fluids

Viscoelastic	Kelvin material, Maxwell material	"Parallel" linearstic combination of elastic and viscous effects ^[1]	Some lubricants, whipped cream, Silly Putty
Time Dependent Viscosity	Rheopecty	Apparent viscosity increases with duration of stress	printer ink, gypsum paste
	Thixotropic	Apparent viscosity decreases with duration of stress ^[1]	Yogurt, xanthan gum solutions, aqueous iron oxide gels, gelatin gels, pectin gels, synovial fluid, hydrogenated castor oil, some clays (including bentonite, and montmorillonite), carbon black suspension in molten tire rubber, some drilling muds, many paints, many floc suspensions, many colloidal suspensions
Time-independent viscosity	Shear thickening (dilatant)	Apparent viscosity increases with increased stress ^[2]	Suspensions of corn starch in water, sand in water
	Shear thinning (pseudoplastic)	Apparent viscosity decreases with increased stress ^{[3][4]}	Nail polish, whipped cream, ketchup, molasses, syrups, paper pulp in water, latex paint, ice, blood, some silicone oils, some silicone coatings
	Generalized Newtonian fluids	Viscosity is constant. Stress depends on normal and shear strain rates and also the pressure applied on it	Blood plasma, custard, water

2.3 Newtonian Fluids and Non-Newtonian Fluids

- Three types of time-independent fluids can be defined:

1. Pseudoplastic (Shear Thinning Fluids)

The plot of shear stress versus velocity gradient lies above the straight, constant sloped line for Newtonian fluids, as shown in Fig. 2.2. The curve begins steeply, indicating a high apparent viscosity. Then the slope decreases with increasing velocity gradient. **decrease in viscosity with increasing shear stress.**

Eg. Ketchup, latex, inks, Greases, mayonnaise.



2.3 Newtonian Fluids and Non-Newtonian Fluids

2. **Dilatant Fluids(Shear Thickening)** The plot of shear stress versus velocity gradient lies below the straight line for Newtonian fluids. The curve begins with a low slope, indicating a low apparent viscosity. Then, the slope increases with increasing velocity gradient. These fluids exhibit an increase in viscosity with increasing shear stress

Eg. Starch in water, Wet beach , TiO_2

3. **Bingham Fluids** Sometimes called *plug-flow fluids*, These fluids behave as solids until an initial yield stress τ_0 is exceeded. Beyond τ_0 , Bingham plastics behave like Newtonian fluids, as illustrated in Fig. 2.2. Once flow starts, there is an essentially linear slope to the curve indicating a constant apparent viscosity.

Eg. Chocolate, drilling muds, toothpaste , soap

2. Viscosity of Fluids

Time-dependent FLUIDS

- Rheopectic fluid

A shear stress that increases with time gives the rheopectic fluid a constant strain rate. Apparent viscosity increases with duration of stress. **EX: A gypsum suspension**



- Thixotropic fluids

- These fluids behave in a manner opposite to rheopectic fluids. A shear stress that decreases with time gives a thixotropic fluid a constant strain rate. Apparent viscosity decreases with duration of stress

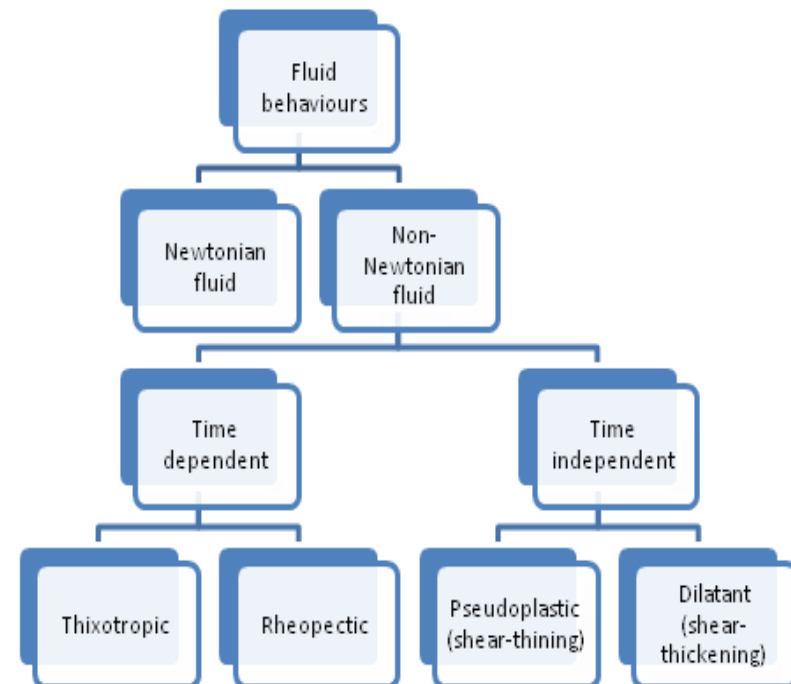
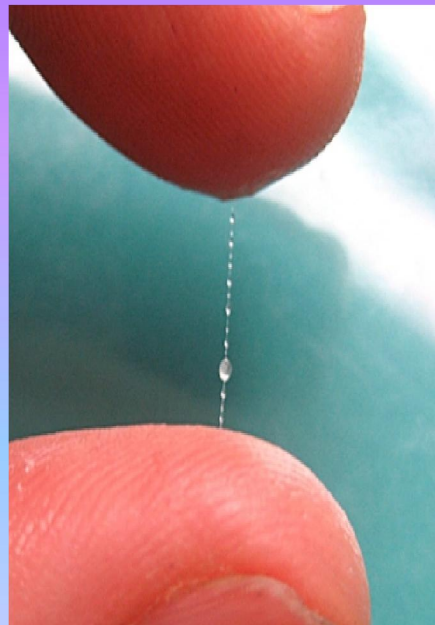
EX: Fast-drying paints



2. Viscosity of Fluids

Viscoelastic Fluids

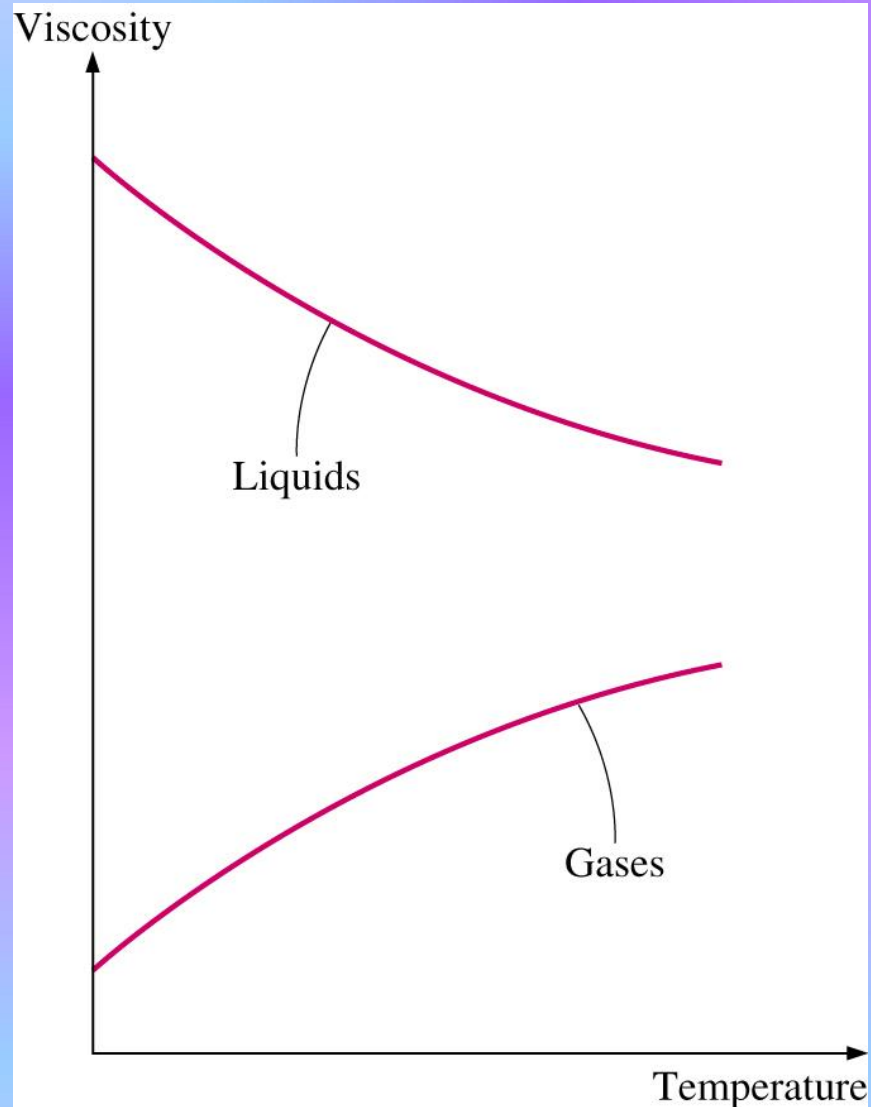
- Such fluids show both **elastic and viscous properties**. They partly recover elastically from deformations caused during the flow.
- EX: Flour dough



2.4 Variation of Viscosity with Temperature

- Table 2.3 shows the viscosity for different fluids.

Fluid	Temperature (°C)	Dynamic Viscosity (N·s/m ² or Pa·s)
Water	20	1.0×10^{-3}
Gasoline	20	3.1×10^{-4}
SAE 30 oil	20	3.5×10^{-1}
SAE 30 oil	80	1.9×10^{-2}



The viscosity of liquids decreases and the viscosity of gases increase with temperature.

As the temperature increases, the viscosities of all liquids decrease, while the viscosities of all gases increase.

2. Viscosity of Fluids

Relation between Viscosity and Temperature

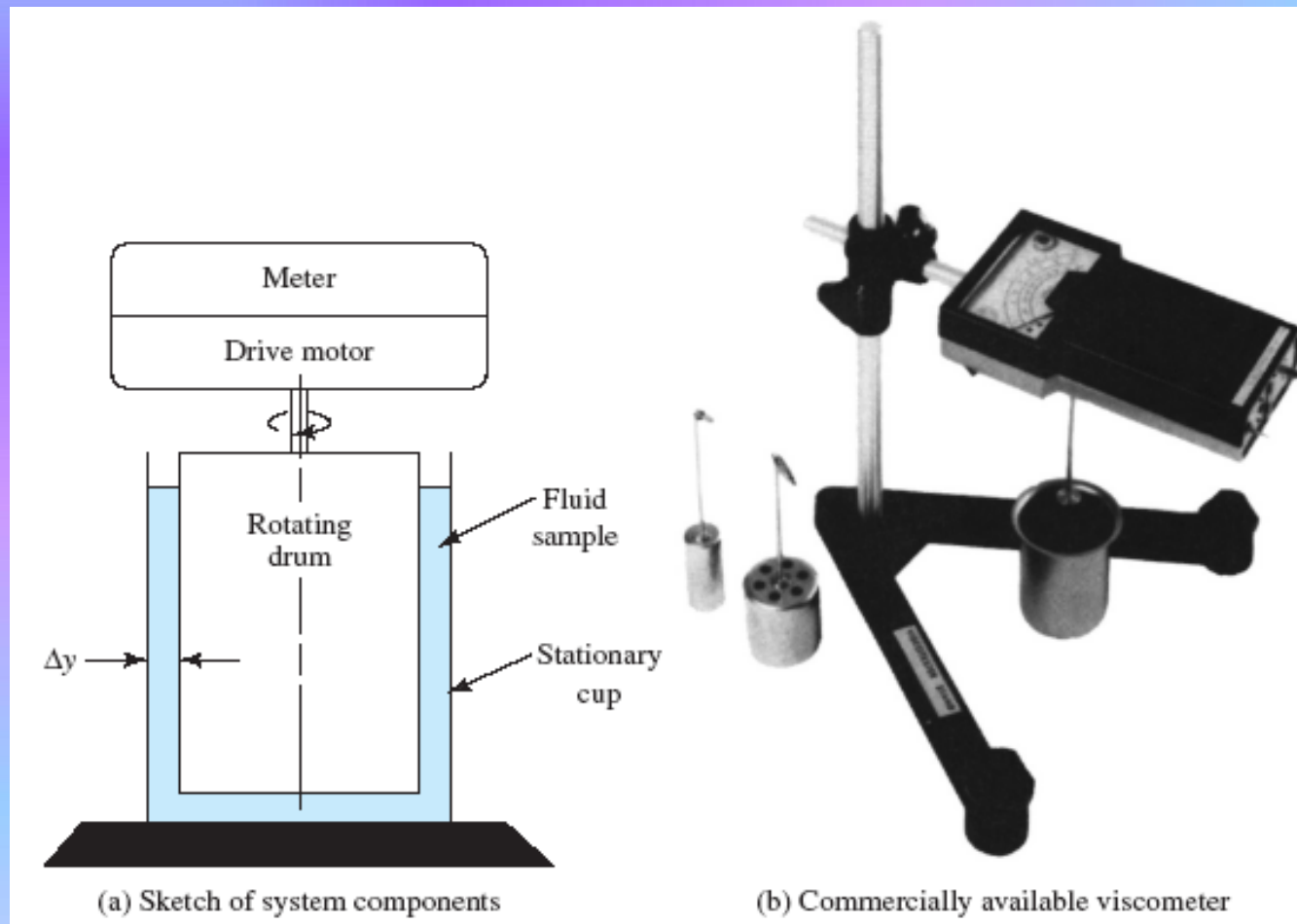
This is because **the force of cohesion, which diminishes with temperature, predominates with liquids**, while with gases the predominating factor **is the interchange of molecules between the layers of different velocities**. Thus a rapidly moving gas molecule shifting into a slower moving layer tends to speed up the latter. And a slow-moving molecule entering a faster-moving layer tends to slow down the faster-moving layer. This **molecular interchange sets** up a shear, or produces a friction force between adjacent layers. At higher temperatures molecular activity increases, so causing the **viscosity of gases to increase with temperature**.

2.5 Viscosity Measurement

- Devices for characterizing the flow behavior of liquids are called *viscometers* or *rheometers*.
- ASTM International generates standards for viscosity measurement and reporting.

2.5.1 Rotating Drum Viscometer

- Fig 2.4 shows the rotating-drum viscometer.



2.5.1 Rotating Drum Viscometer

- The apparatus shown in Fig. 2.4(a) measures viscosity by the definition of dynamic viscosity given in Eq. (2–2), which we can write in the form

$$\mu = \tau / (\Delta v / \Delta y)$$

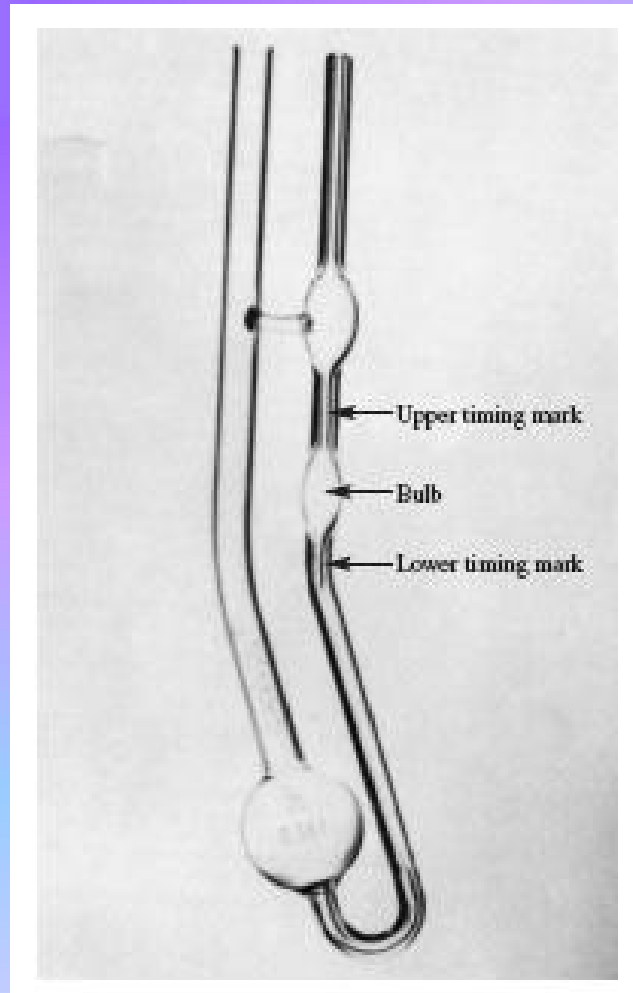
- The dynamic viscosity of the fluid can be computed from the simple equation

$$\mu = \frac{K}{(n_2/n_1 - 1)}$$

- where n_2 is the speed of the outer tube and n_1 is the speed of the internal rotor. K is a calibration constant provided by the instrument manufacturer.

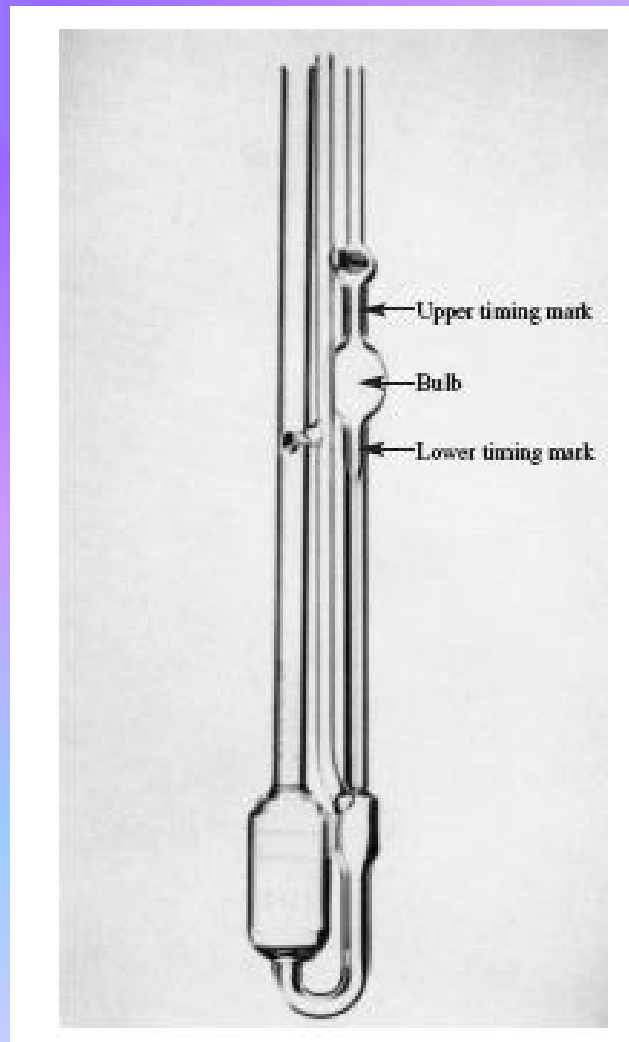
2.5.3 Standard Calibrated Glass Capillary Viscometers

- Fig 2.6 shows the Cannon–Fenske routine viscometer.



2.5.3 Standard Calibrated Glass Capillary Viscometers

- Fig 2.7 shows the Ubbelohde viscometer.



2.5.3 Standard Calibrated Glass Capillary Viscometers

- The kinematic viscosity is computed by multiplying the flow time by the calibration constant of the viscometer supplied by the vendor.
- The viscosity unit used in these tests is the centistoke (cSt), which is equivalent to mm^2/s .

2.5.4 Falling-Ball Viscometer

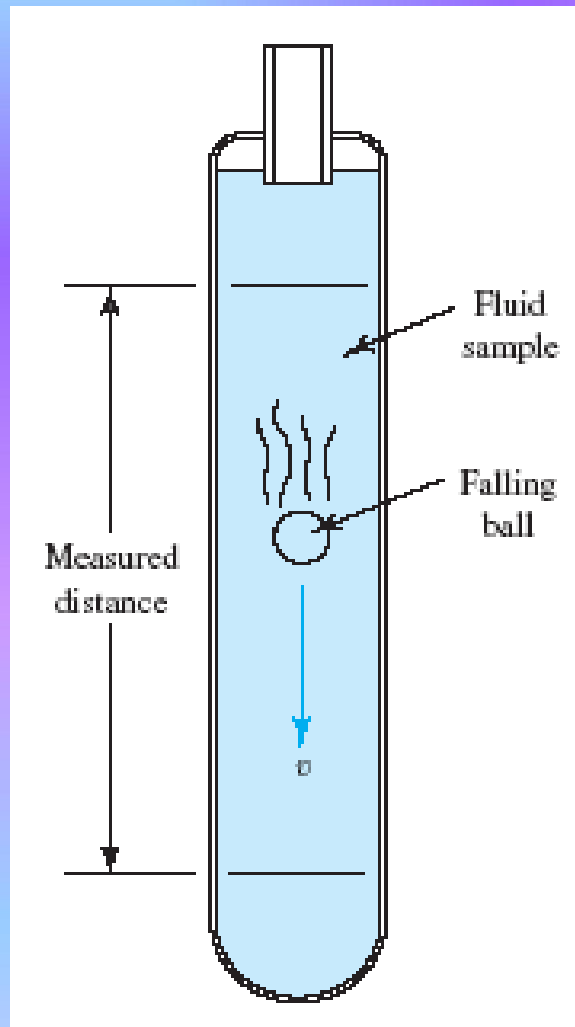
- As a body falls in a fluid under the influence of gravity only, it will accelerate until the downward force (its weight) is just balanced by the buoyant force and the viscous drag force acting upward.
- Its velocity at that time is called the *terminal velocity*.

2.5.4 Falling-Ball Viscometer



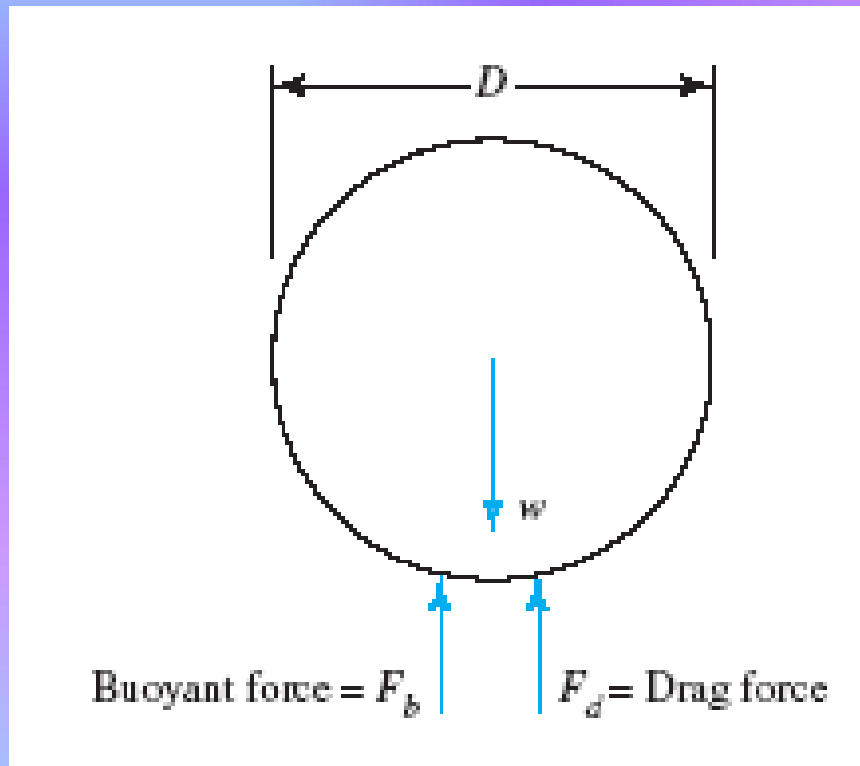
The kinematic viscosity bath for holding standard calibrated glass capillary viscometers.

2.5.4 Falling-Ball Viscometer



The falling-ball viscometer.

2.5.4 Falling-Ball Viscometer



The free-body diagram of a ball in a falling-ball viscometer.

2.5.4 Falling-Ball Viscometer

- Figure 2.10 shows a free-body diagram of the ball, where w is the weight of the ball, F_b is the buoyant force, and F_d is the viscous drag force on the ball.
- Therefore, we have

$$w - F_b - F_d = 0. \quad (2-6)$$

- If γ_s is the specific weight of the sphere, γ_f is the specific weight of the fluid, V is the volume of the sphere, and D is the diameter of the sphere, we have

$$w = \gamma_s V = \gamma_s \pi D^3 / 6 \quad (2-7)$$

$$F_b = \gamma_f V = \gamma_f \pi D^3 / 6 \quad (2-8)$$

2.5.4 Falling-Ball Viscometer

- For very viscous fluids and a small velocity, the drag force on the sphere is

$$F_d = 3\pi\mu vD \quad (2-9)$$

- Equation (2-6) then becomes

$$\mu = \frac{(\gamma_s - \gamma_f)D^2}{18v} \quad (2-10)$$

2.6 SAE Viscosity Grade

SAE Viscosity Grade	Low Temperature—Dynamic Viscosity		High-Temperature		
	Cranking Condition* (cP) Max. at (°C)	Pumping Condition# (cP) Max. at (°C)	Kinematic Viscosity at 100°C (cSt) ⁺		High-Temperature, High-Shear-Rate Dynamic Viscosity [◇] at 150°C (cP) Min.
			Min.	Max.	
0W	6200 at -35	60 000 at -40	3.8	—	—
5W	6600 at -30	60 000 at -35	3.8	—	—
10W	7000 at -25	60 000 at -30	4.1	—	—
15W	7000 at -20	60 000 at -25	5.6	—	—
20W	9500 at -15	60 000 at -20	5.6	—	—
25W	13 000 at -10	60 000 at -15	9.3	—	—
20	—	—	5.6	<9.3	2.6
30	—	—	9.3	<12.5	2.9
40	—	—	12.5	<16.3	2.9 [⊥]
40	—	—	12.5	<16.3	3.7 [⌈]
50	—	—	16.3	<21.9	3.7
60	—	—	21.9	<26.1	3.7

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Note: 1 cP = 1 mPa · s ; 1 cSt = 1 mm²/s

* Using ASTM Standard D 5293

Using ASTM D 4684

⁺ Using ASTM D 445

[◇] Using ASTM D 4683, D 4741, or D 5481

[⊥] When used in these multiviscosity grades: 0W-40, 5W-40, 10W-40

[⌈] When used in single-grade SAE 40 and in these multiviscosity grades: 15W-40, 20W-40, 25W-40

2.6 SAE Viscosity Grade

SAE Viscosity Grade	Maximum Temperature for Dynamic Viscosity of 150 000 cP* (°C)	Kinematic Viscosity at 100°C (cSt) [‡]	
		Min.	Max.
70W	−55	4.1	—
75W	−40	4.1	—
80W	−26	7.0	—
85W	−12	11.0	—
80	—	7.0	<11.0
85	—	11.0	<13.5
90	—	13.5	<24.0
140	—	24.0	<41.0
250	—	41.0	—

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Note: 1 cP = 1 mPa·s; 1 cSt = 1 mm²/s

*Using ASTM D 2983

[‡]Using ASTM D 445

2.7 ISO Viscosity Grade

- The standard designation includes the prefix ISO VG followed by a number representing the nominal kinematic viscosity in cSt for a temperature of 40°C.
- Table 2.6 gives the data.

Grade ISO VG	Kinematic Viscosity at 40°C (cSt) or (mm ² /s)		
	Nominal	Minimum	Maximum
2	2.2	1.98	2.40
3	3.2	2.88	3.52
5	4.6	4.14	5.06
7	6.8	6.12	7.48
10	10	9.00	11.0
15	15	13.5	16.5
22	22	19.8	24.2
32	32	28.8	35.2
46	46	41.4	50.6
68	68	61.2	74.8
100	100	90.0	110
150	150	135	165
220	220	198	242
320	320	288	352
460	460	414	506
680	680	612	748
1000	1000	900	1100
1500	1500	1350	1650
2200	2200	1980	2420
3200	3200	2880	3520

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Quiz Problem

Initially when 1000 mL of water at 10°C are poured into a glass cylinder the height of the water column is 1000 mm. The water and its container are heated to 70°C. Assuming no evaporation. What then will be the depth of the water column if the coefficient of thermal expansion for the glass is 3.8×10^{-6} mm/mm per °C?

Given: Water Density at 10°C = 999.7 kg/m³

Water Density at 70°C = 977.8 kg/m³